Exercise: Relationship between $D(p||q)$ and χ^2 statistic

(Source: (**?**, Q12.2).)

Show that, if $p(x) \approx q(x)$, then

$$\mathbb{KL}\left(p||q\right) \approx \frac{1}{2}\chi^2 \tag{1}$$

where

$$\chi^2 = \sum_x \frac{(p(x) - q(x))^2}{q(x)} \tag{2}$$

Hint: write

$$p(x) = \Delta(x) + q(x) \tag{3}$$

$$\frac{p(x)}{q(x)} = 1 + \frac{\Delta(x)}{q(x)} \tag{4}$$

and use the Taylor series expansion for $\log(1+x)$.

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots \tag{5}$$

for $-1 < x \leq 1$.