Exercise: Derivation of Q function for HMM

Show that the expected complete data log likelihood for an HMM is given by

$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{old}) = \sum_{k=1}^{K} \mathbb{E}\left[N_k^1\right] \log \pi_k + \sum_{j=1}^{K} \sum_{k=1}^{K} \mathbb{E}\left[N_{jk}\right] \log A_{jk} \quad (1)$$

$$+ \sum_{n=1}^{N} \sum_{t=1}^{T_n} \sum_{k=1}^{K} p(z_t = k | \mathbf{x}_n, \boldsymbol{\theta}^{old}) \log p(\mathbf{x}_{n,t} | \boldsymbol{\theta}_k) \quad (2)$$

where T_n is the length of sequence n.