Exercise: James Stein estimator for Gaussian means

Consider the 2 stage model $Y_i|\theta_i \sim \mathcal{N}(\theta_i, \sigma^2)$ and $\theta_i|\mu \sim \mathcal{N}(m_0, \tau_0^2)$. Suppose $\sigma^2 = 500$ is known and we observe the following 6 data points: 1505, 1528, 1564, 1498, 1600, 1470.

1. Find the ML-II estimates of m_0 and τ_0^2.
2. Find the posterior estimates $\mathbb{E}[\theta_i|y_i, m_0, \tau_0]$ and $\mathbb{V}[\theta_i|y_i, m_0, \tau_0]$ for $i = 1$. (The other terms, $i = 2:6$, are computed similarly.)
3. Give a 95% credible interval for $p(\theta_i|y_i, m_0, \tau_0)$ for $i = 1$. Do you trust this interval (assuming the Gaussian assumption is reasonable)? i.e. is it likely to be too large or too small, or just right?
4. What do you expect would happen to your estimates if σ^2 were much smaller (say $\sigma^2 = 1$)? You do not need to compute the numerical answer; just briefly explain what would happen qualitatively, and why.