Exercise: Derivation of the structured mean field updates for FHMM

Derive the updates below for the structured mean field approximation for factorial HMMs:

$$\boldsymbol{\xi}_{tm} = \exp\left(\mathbf{W}_{m}^{\top} \boldsymbol{\Sigma}^{-1} \tilde{\mathbf{x}}_{tm} - 1/2 \boldsymbol{\delta}_{m}\right)$$
 (1)

$$\boldsymbol{\delta}_m \triangleq \operatorname{diag}(\mathbf{W}_m^{\top} \boldsymbol{\Sigma}^{-1} \mathbf{W}_m) \tag{2}$$

$$\boldsymbol{\xi}_{tm} = \exp\left(\mathbf{W}_{m}^{\top} \boldsymbol{\Sigma}^{-1} \tilde{\mathbf{x}}_{tm} - 1/2 \boldsymbol{\delta}_{m}\right)$$

$$\boldsymbol{\delta}_{m} \triangleq \operatorname{diag}(\mathbf{W}_{m}^{\top} \boldsymbol{\Sigma}^{-1} \mathbf{W}_{m})$$

$$\tilde{\mathbf{x}}_{tm} \triangleq \mathbf{x}_{t} - \sum_{\ell \neq m}^{M} \mathbf{W}_{\ell} \mathbb{E}\left[\mathbf{z}_{t,\ell}\right]$$
(3)