Exercise: A measure of correlation (normalized mutual information)

(Source: (?, Q2.20).)

Let X and Y be discrete random variables which are identically distributed (so H(X) = H(Y)) but not necessarily independent. Define

$$r = 1 - \frac{H(Y|X)}{H(X)} \tag{1}$$

- 1. Show $r = \frac{I(X,Y)}{H(X)}$
- 2. Show $0 \le r \le 1$
- 3. When is r = 0?
- 4. When is r = 1?