

Exercise: Graphcuts for alpha-beta swap

(Source: Ex. 13.15 of (3)). Show how the optimal alpha-beta swap can be found by running min-cut on an appropriately constructed graph. More precisely,

1. Define a set of binary variables t_1, \dots, t_n such that $t_i = 0$ means $x'_i = \alpha$, $t_i = 1$ if $x'_i = \beta$, and $x'_i = x_i$ is unchanged if $x_i \neq \alpha$ and $x_i \neq \beta$.
2. Define an energy function over the new variables such that $E'(\mathbf{t}) = E(\mathbf{x}) + \text{const.}$
3. Show that E' is submodular if E is a semimetric.