Exercise: Inference in a simple Bayes net for fish classification

(Source: (**?**).)

Consider the Bayes net shown in Figure 1. Here, the nodes represent the following variables

$$X_1 \in \{\text{winter, spring, summer, autumn}\},\ X_2 \in \{\text{salmon, sea bass}\} \tag{1}$$

$$X_3 \in \{\text{light, medium, dark}\},\ X_4 \in \{\text{wide, thin}\} \tag{2}$$

The corresponding conditional probability tables are

$$p(x_1) = \begin{pmatrix} .25 & .25 & .25 & .25 \end{pmatrix},\ p(x_2|x_1) = \begin{pmatrix} .9 & .1 \\ .3 & .7 \\ .4 & .6 \\ .8 & .2 \end{pmatrix} \tag{3}$$

$$p(x_3|x_2) = \begin{pmatrix} .33 & .33 & .34 \\ .8 & .1 & .1 \end{pmatrix},\ p(x_4|x_2) = \begin{pmatrix} .4 & .6 \\ .95 & .05 \end{pmatrix} \tag{4}$$

Note that in $p(x_4|x_2)$, the rows represent x_2 and the columns x_4 (so each row sums to one and represents the child of the CPD). Thus $p(x_4 = \text{thin}|x_2 = \text{sea bass}) = 0.05$, $p(x_4 = \text{thin}|x_2 = \text{salmon}) = 0.6$, etc.

Answer the following queries. You may use matlab or do it by hand. In either case, show your work.

1. Suppose the fish was caught on December 20 — the end of autumn and the beginning of winter — and thus let $p(x_1) = (.5,\ 0,\ 0,\ .5)$ instead of the above prior. (This is called **soft evidence**, since we do not know the exact value of X_1, but we have a distribution over it.) Suppose the lightness has not been measured but it is known that the fish is thin. Classify the fish as salmon or sea bass.

2. Suppose all we know is that the fish is thin and medium lightness. What season is it now, most likely? Use $p(x_1) = \begin{pmatrix} .25 & .25 & .25 & .25 \end{pmatrix}$

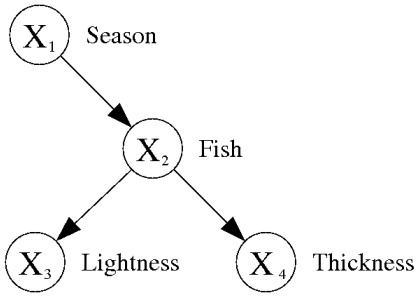

Figure 1: Fishnet.