Exercise: Variational EM for binary FA with sigmoid link

Consider the binary FA model:

$$p(\mathbf{x}_i|\mathbf{z}_i,\boldsymbol{\theta}) = \prod_{j=1}^{D} \mathrm{Ber}(x_{ij}|\sigma(\mathbf{w}_j^T\mathbf{z}_i+\beta_j)) = \prod_{j=1}^{D} \mathrm{Ber}(x_{ij}|\sigma(\eta_{ij})) \tag{1}$$

$$\boldsymbol{\eta}_i = \tilde{\mathbf{W}}\tilde{\mathbf{z}}_i \tag{2}$$

$$\tilde{\mathbf{z}}_i \triangleq (\mathbf{z}_i;1) \tag{3}$$

$$\tilde{\mathbf{W}} \triangleq (\mathbf{W},\boldsymbol{\beta}) \tag{4}$$

$$p(\mathbf{z}_i) = \mathcal{N}(\mathbf{0},\mathbf{I}) \tag{5}$$

Derive an EM algorithm to fit this model, using the Jaakkola-Jordan bound to the logistic function

$$\sigma(x) \geq \sigma(\xi)\exp\left[(x-\xi)/2 - \lambda(\xi)(x^2-\xi^2)\right] \tag{6}$$

where ξ is a variational parameter.