

Exercise: Conditional independence iff joint factorizes

In the text we said $X \perp Y|Z$ iff

$$p(x, y|z) = p(x|z)p(y|z) \tag{1}$$

for all x, y, z such that $p(z) > 0$. Now prove the following alternative definition: $X \perp Y|Z$ iff there exist functions g and h such that

$$p(x, y|z) = g(x, z)h(y, z) \tag{2}$$

for all x, y, z such that $p(z) > 0$.