Exercise: Conditional independence

(Source: Koller.)

1. Let $H \in \{1, \ldots, K\}$ be a discrete random variable, and let e_1 and e_2 be the observed values of two other random variables E_1 and E_2. Suppose we wish to calculate the vector

$$\vec{P}(H|e_1, e_2) = (P(H = 1|e_1, e_2), \ldots, P(H = K|e_1, e_2))$$

   Which of the following sets of numbers are sufficient for the calculation?

   (a) $P(e_1, e_2)$, $P(H)$, $P(e_1|H)$, $P(e_2|H)$

   (b) $P(e_1, e_2)$, $P(H)$, $P(e_1, e_2|H)$

   (c) $P(e_1|H)$, $P(e_2|H)$, $P(H)$

2. Now suppose we now assume $E_1 \perp E_2|H$ (i.e., E_1 and E_2 are conditionally independent given H). Which of the above 3 sets are sufficient now?

Show your calculations as well as giving the final result. Hint: use Bayes rule.