

Exercise: Reducing elastic net to lasso

Define

$$J_1(\mathbf{w}) = \|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 + \lambda_2 \|\mathbf{w}\|^2 + \lambda_1 \|\mathbf{w}\|_1 \quad (1)$$

and

$$J_2(\mathbf{w}) = \|\tilde{\mathbf{y}} - \tilde{\mathbf{X}}\mathbf{w}\|^2 + c\lambda_1 \|\mathbf{w}\|_1 \quad (2)$$

where $c = (1 + \lambda_2)^{-1/2}$ and

$$\tilde{\mathbf{X}} = c \begin{pmatrix} \mathbf{X} \\ \sqrt{\lambda_2} \mathbf{I}_d \end{pmatrix}, \quad \tilde{\mathbf{y}} = \begin{pmatrix} \mathbf{y} \\ \mathbf{0}_{d \times 1} \end{pmatrix} \quad (3)$$

Show

$$\operatorname{argmin} J_1(\mathbf{w}) = c(\operatorname{argmin} J_2(\mathbf{w})) \quad (4)$$

i.e.

$$J_1(c\mathbf{w}) = J_2(\mathbf{w}) \quad (5)$$

and hence that one can solve an elastic net problem using a lasso solver on modified data.