

**Exercise: Derivation of Fisher's linear discriminant**

Show that the maximum of  $J(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S}_B \mathbf{w}}{\mathbf{w}^T \mathbf{S}_W \mathbf{w}}$  is given by  $\mathbf{S}_B \mathbf{w} = \lambda \mathbf{S}_W \mathbf{w}$

where  $\lambda = \frac{\mathbf{w}^T \mathbf{S}_B \mathbf{w}}{\mathbf{w}^T \mathbf{S}_W \mathbf{w}}$ . Hint: recall that the derivative of a ratio of two scalars is given by  $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'g - fg'}{g^2}$ , where  $f' = \frac{d}{dx} f(x)$  and  $g' = \frac{d}{dx} g(x)$ . Also, recall that  $\frac{d}{d\mathbf{x}} \mathbf{x}^T \mathbf{A} \mathbf{x} = (\mathbf{A} + \mathbf{A}^T) \mathbf{x}$ .