Exercise: Uncorrelated and Gaussian does not imply independent unless *jointly* Gaussian

Let $X \sim \mathcal{N}(0,1)$ and $Y = WX$, where $p(W=-1) = p(W=1) = 0.5$. It is clear that X and Y are not independent, since Y is a function of X.

1. Show $Y \sim \mathcal{N}(0,1)$.

2. Show $\text{Cov}[X,Y] = 0$. Thus X and Y are uncorrelated but dependent, even though they are Gaussian. Hint: use the definition of covariance

$$\text{Cov}[X,Y] = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y] \tag{1}$$

   and the **rule of iterated expectation**

$$\mathbb{E}[XY] = \mathbb{E}[\mathbb{E}[XY|W]] \tag{2}$$