

Exercise: ML estimator $\hat{\sigma}_{\text{mle}}^2$ is biased

Show that $\hat{\sigma}_{MLE}^2 = \frac{1}{N} \sum_{n=1}^N (x_n - \hat{\mu})^2$ is a biased estimator of σ^2 , i.e., show

$$\mathbf{E}_{X_1, \dots, X_n \sim \mathcal{N}(\mu, \sigma)}[\hat{\sigma}^2(X_1, \dots, X_n)] \neq \sigma^2$$

Hint: note that X_1, \dots, X_N are independent, and use the fact that the expectation of a product of independent random variables is the product of the expectations.