

Exercise: Normalization constant for a multidimensional Gaussian

Prove that the normalization constant for a d -dimensional Gaussian is given by

$$(2\pi)^{d/2} |\Sigma|^{1/2} = \int \exp(-1/2(\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{x} - \boldsymbol{\mu})) d\mathbf{x} \quad (1)$$

Hint: diagonalize Σ and use the fact that $|\Sigma| = \prod_i \lambda_i$ to write the joint pdf as a product of d one-dimensional Gaussians in a transformed coordinate system. (You will need the change of variables formula.) Finally, use the normalization constant for univariate Gaussians.