Exercise: Gaussian posterior credible interval

(Source: DeGroot.)

Let $X \sim \mathcal{N}(\mu, \sigma^2 = 4)$ where μ is unknown but has prior $\mu \sim \mathcal{N}(\mu_0, \sigma_0^2 = 9)$. The posterior after seeing n samples is $\mu \sim \mathcal{N}(\mu_n, \sigma_n^2)$. (This is called a credible interval, and is the Bayesian analog of a confidence interval.) How big does n have to be to ensure

$$p(\ell \leq \mu_n \leq u|D) \geq 0.95 \tag{1}$$

where (ℓ, u) is an interval (centered on μ_n) of width 1 and D is the data? Hint: recall that 95% of the probability mass of a Gaussian is within $\pm 1.96\sigma$ of the mean.