Exercise: Derivation of information form formulae for marginalizing and conditioning an MVN

Derive the equations for the marginal and conditional of an MVN in information form, i.e., show that

$$p(\mathbf{x}_{2}) = \mathcal{N}_{c}(\mathbf{x}_{2}|\boldsymbol{\xi}_{2} - \boldsymbol{\Lambda}_{21}\boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{\xi}_{1}, \boldsymbol{\Lambda}_{22} - \boldsymbol{\Lambda}_{21}\boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{\Lambda}_{12})$$
(1)
$$p(\mathbf{x}_{1}|\mathbf{x}_{2}) = \mathcal{N}_{c}(\mathbf{x}_{1}|\boldsymbol{\xi}_{1} - \boldsymbol{\Lambda}_{12}\mathbf{x}_{2}, \boldsymbol{\Lambda}_{11})$$
(2)

$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathcal{N}_c(\mathbf{x}_1|\boldsymbol{\xi}_1 - \boldsymbol{\Lambda}_{12}\mathbf{x}_2, \boldsymbol{\Lambda}_{11})$$
 (2)