Exercise: Deriving the inverse gamma density

Let $X \sim \mathrm{Ga}(a, b)$, i.e.

$$\mathrm{Ga}(x|a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-xb} \tag{1}$$

Let $Y = 1/X$. Show that $Y \sim \mathrm{IG}(a, b)$, i.e.,

$$\mathrm{IG}(x|\text{shape} = a, \text{scale} = b) = \frac{b^a}{\Gamma(a)} x^{-(a+1)} e^{-b/x} \tag{2}$$

Hint: use the change of variables formula.