Exercise: Deriving E step for GSM prior

Show that

$$\mathbb{E}\left[\frac{1}{\tau_j^2}|w_j\right] \quad = \quad \frac{\pi'(w_j)}{|w_j|} \tag{1}$$

where $\pi(w_j) = -\log p(w_j)$ and $p(w_j) = \int \mathcal{N}(w_j|0, \tau_j^2)p(\tau_j^2)d\tau_j^2$.

Hint 1:

$$\begin{aligned}
\frac{1}{\tau_j^2}\mathcal{N}(w_j|0, \tau_j^2) &\propto \frac{1}{\tau_j^2}\exp(-\frac{w_j^2}{2\tau_j^2}) && (2)\\
&= \frac{-1}{|w_j|}\frac{-2w_j}{2\tau_j^2}\exp(-\frac{w_j^2}{2\tau_j^2}) && (3)\\
&= \frac{-1}{|w_j|}\frac{d}{d|w_j|}\mathcal{N}(w_j|0, \tau_j^2) && (4)
\end{aligned}$$

Hint 2:

$$\frac{d}{d|w_j|}p(w_j) = \frac{1}{p(w_j)}\frac{d}{d|w_j|}\log p(w_j) \tag{5}$$