

### Exercise: Multi-output linear regression

(Source: Jaakkola.)

Consider a linear regression model with a 2 dimensional response vector  $\mathbf{y}_i \in \mathbb{R}^2$ . Suppose we have some binary input data,  $x_i \in \{0, 1\}$ . The training data is as follows:

x	y
0	$(-1, -1)^T$
0	$(-1, -2)^T$
0	$(-2, -1)^T$
1	$(1, 1)^T$
1	$(1, 2)^T$
1	$(2, 1)^T$

Let us embed each  $x_i$  into 2d using the following basis function:

$$\phi(0) = (1, 0)^T, \quad \phi(1) = (0, 1)^T \quad (1)$$

The model becomes

$$\hat{\mathbf{y}} = \mathbf{W}^T \phi(x) \quad (2)$$

where  $\mathbf{W}$  is a  $2 \times 2$  matrix. Compute the MLE for  $\mathbf{W}$  from the above data.