

Exercise: A mixture of conjugate priors is conjugate

Consider a mixture prior

$$p(\theta) = \sum_k p(h = k)p(\theta|z = k) \tag{1}$$

where each $p(\theta|z = k)$ is conjugate to the likelihood. Prove that this is a conjugate prior.