Exercise: Message passing on a tree

Consider the DGM in Figure 1, which represents the following fictitious biological model. Each G_i represents the genotype of a person: $G_i = 1$ if they have a healthy gene and $G_i = 2$ if they have an unhealthy gene. G_2 and G_3 may inherit the unhealthy gene from their parent G_1. $X_i \in \mathbb{R}$ is a continuous measure of blood pressure, which is low if you are healthy and high if you are unhealthy. We define the CPDs as follows

$$p(G_1) = [0.5, 0.5] \tag{1}$$

$$p(G_2|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix} \tag{2}$$

$$p(G_3|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix} \tag{3}$$

$$p(X_i|G_i = 1) = \mathcal{N}(X_i|\mu = 50, \sigma^2 = 10) \tag{4}$$

$$p(X_i|G_i = 2) = \mathcal{N}(X_i|\mu = 60, \sigma^2 = 10) \tag{5}$$

The meaning of the matrix for $p(G_2|G_1)$ is that $p(G_2 = 1|G_1 = 1) = 0.9$, $p(G_2 = 1|G_1 = 2) = 0.1$, etc.

1. Suppose you observe $X_2 = 50$, and X_1 is unobserved. What is the posterior belief on G_1, i.e., $p(G_1|X_2 = 50)$?
2. Now suppose you observe $X_2 = 50$ amd $X_3 = 50$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.
3. Now suppose $X_2 = 60$, $X_3 = 60$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.
4. Now suppose $X_2 = 50$, $X_3 = 60$. What is $p(G_1|X_1, X_2)$? Explain your answer intuitively.

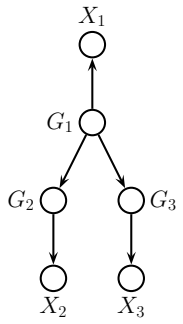

Figure 1: A simple DAG representing inherited diseases.