

Exercise: Newsvendor problem

Consider the following classic problem in decision theory / economics. Suppose you are trying to decide how much quantity Q of some product (e.g., newspapers) to buy to maximize your profits. The optimal amount will depend on how much demand D you think there is for your product, as well as its cost to you C and its selling price P . Suppose D is unknown but has pdf $f(D)$ and cdf $F(D)$. We can evaluate the expected profit by considering two cases: if $D > Q$, then we sell all Q items, and make profit $\pi = (P - C)Q$; but if $D < Q$, we only sell D items, at profit $(P - C)D$, but have wasted $C(Q - D)$ on the unsold items. So the expected profit if we buy quantity Q is

$$E\pi(Q) = \int_Q^\infty (P - C)Qf(D)dD + \int_0^Q (P - C)Df(D)dD - \int_0^Q C(Q - D)f(D)dD \quad (1)$$

Simplify this expression, and then take derivatives wrt Q to show that the optimal quantity Q^* (which maximizes the expected profit) satisfies

$$F(Q^*) = \frac{P - C}{P} \quad (2)$$