Exercise: Orthogonal matrices

1. A rotation in 3d by angle α about the z axis is given by the following matrix:

$$\mathbf{R}(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Prove that $\mathbf{R}$ is an orthogonal matrix, i.e., $\mathbf{R}^T\mathbf{R} = \mathbf{I}$, for any α.

2. What is the only eigenvector $\mathbf{v}$ of $\mathbf{R}$ with an eigenvalue of 1.0 and of unit norm (i.e., $||\mathbf{v}||^2 = 1$)? (Your answer should be the same for any α.) Hint: think about the geometrical interpretation of eigenvectors.