

Exercise: Pairwise independence does not imply mutual independence

We say that two random variables are pairwise independent if

$$p(X_2|X_1) = p(X_2) \quad (1)$$

and hence

$$p(X_2, X_1) = p(X_1)p(X_2|X_1) = p(X_1)p(X_2) \quad (2)$$

We say that n random variables are mutually independent if

$$p(X_i|X_S) = p(X_i) \quad \forall S \subseteq \{1, \dots, n\} \setminus \{i\} \quad (3)$$

and hence

$$p(X_{1:n}) = \prod_{i=1}^n p(X_i) \quad (4)$$

Show that pairwise independence between all pairs of variables does not necessarily imply mutual independence. It suffices to give a counter example.