Exercise: Optimal proposal for particle filtering with linear-Gaussian measurement model

Consider a state-space model of the following form:

$$\mathbf{z}_t = f_t(\mathbf{z}_{t-1}) + \mathcal{N}(\mathbf{0}, \mathbf{Q}_{t-1}) \quad (1)$$

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{z}_t + \mathcal{N}(\mathbf{0}, \mathbf{R}_t) \quad (2)$$

Derive expressions for $p(\mathbf{z}_t|\mathbf{z}_{t-1}, \mathbf{y}_t)$ and $p(\mathbf{y}_t|\mathbf{z}_{t-1})$, which are needed to compute the optimal (minimum variance) proposal distribution. Hint: use Bayes rule for Gaussians.