

Exercise: Sensor fusion with known variances in 1d

Suppose we have two sensors with known (and different) variances v_1 and v_2 , but unknown (and the same) mean μ . Suppose we observe n_1 observations $y_i^{(1)} \sim \mathcal{N}(\mu, v_1)$ from the first sensor and n_2 observations $y_i^{(2)} \sim \mathcal{N}(\mu, v_2)$ from the second sensor. (For example, suppose μ is the true temperature outside, and sensor 1 is a precise (low variance) digital thermosensing device, and sensor 2 is an imprecise (high variance) mercury thermometer.) Let \mathcal{D} represent all the data from both sensors. What is the posterior $p(\mu|\mathcal{D})$, assuming a non-informative prior for μ (which we can simulate using a Gaussian with a precision of 0)? Give an explicit expression for the posterior mean and variance.