

Exercise: Two filter approach to smoothing in HMMs

Assuming that $\Pi_t(i) = p(S_t = i) > 0$ for all i and t , derive a recursive algorithm for updating $r_t(i) = p(S_t = i | \mathbf{x}_{t+1:T})$. Hint: it should be very similar to the standard forwards algorithm, but using a time-reversed transition matrix. Then show how to compute the posterior marginals $\gamma_t(i) = p(S_t = i | \mathbf{x}_{1:T})$ from the backwards filtered messages $r_t(i)$, the forwards filtered messages $\alpha_t(i)$, and the stationary distribution $\Pi_t(i)$.